The Hurst Phenomenon in Atmospheric Turbulence

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Summary

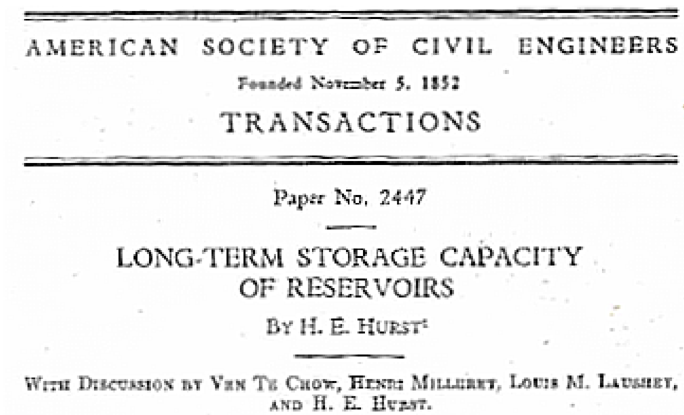
Thanks and acknowledgements

It is an honor to be here, to speak to you.

Thanks to the organizing committee for the invitation.

This talk is largely based on Bianca Luhm's MSc thesis and joint work with M. Chamecki.

1. Introduction



AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

TRANSACTIONS

Paper No. 2447

LONG-TERM STORAGE CAPACITY OF RESERVOIRS

By H. E. Hurst[1]

With Discussion by Ven Te Chow, Henri Milleret, Louis M. Laushey, and H. E. Hurst.

The Hurst Phenomenon is named after H. E. Hurst's "Long-Term Storage Capacity of Reservoirs" (Trans. ASCE 116, 776–808, 1951).

At the beginning of the paper,

> It is thought that the general theory may have other applications than the design of reservoirs for the storage of water.

Hurst had been interested in the design of reservoirs for the Nile River which should be able to meet a certain target demand.

The birth of “Stochastic Hydrology” (computer simulation of streamflows)

In Hurst’s paper, we find a reference to the precise moment of birth of Stochastic Hydrology as a science of simulating river streamflow:

> …The work was extended by the late Charles E. Sudler, M. ASCE., by similar graphical methods. Much of Mr. Sudler’s work was based on artifical records in which information taken from a short record of a stream was extended to a period of 1,000 years by writing, say, 50 annual runoff values on cards and then shuffling and drawing a card from these 1,000 times.

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Maximum yield from a reservoir over a period Δ

If the inflow into a reservoir is $x(t)$, the average over a period Δ (say, 100 years) is

$$\widetilde{x}_\Delta = \frac{1}{\Delta}\int_0^\Delta x(t)\,\mathrm{d}t.$$

(bear with me, atmospheric turbulence will eventually appear). For any length $\delta < \Delta$ starting at $t = 0$, the mean inflow will be

$$\widetilde{x}_\delta = \frac{1}{\delta}\int_0^\delta x(t)\,\mathrm{d}t.$$

For a reservoir to be able to supply $\widetilde{x}_\Delta$ continuously (the maximum that it can), its size must be the *Range*

$$R_\Delta = \max_\delta\left[\delta\left(\widetilde{x}_\delta - \widetilde{x}_\Delta\right)\right] - \min_\delta\left[\delta\left(\widetilde{x}_\delta - \widetilde{x}_\Delta\right)\right].$$

As it is painfully known,

(see the current water shortages in Southeastern Brazil) the sample range depends on the period analyzed [Hurst, 1951, Table 2].

Description	1871-1908	1909-1945	1871-1945
Number of years, Δ	38	37	75
Average annual discharge, Q, milliards m^3	103	83	93
Range of accumulated departures, R, milliards m^3	201	83	476

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Hurst's engineering question was:

Can I predict the range R, given Δ?

2. Theory

Natural extension to incorporate ensemble averaging: **always** at scale Δ and beginning at time t, define:

The sample mean: $$\widetilde{x}_\Delta(t) = \frac{1}{\Delta}\int_t^{t+\Delta} x(t')\,\mathrm{d}t',$$

The sample stdev: $$s^2_\Delta(t) = \frac{1}{\Delta}\int_t^{t+\Delta} \left[x(t') - \widetilde{x}_\Delta(t)\right]^2\mathrm{d}t',$$

The departure: $$v_\Delta(t) = \left[\widetilde{x}_\Delta(t) - \langle x\rangle\right]^2,$$

The range: $$R_\Delta(t) = \max_{0\le\delta\le\Delta}\left[\delta\left(\widetilde{x}_\delta(t) - \widetilde{x}_\Delta(t)\right)\right] - \min_{0\le\delta\le\Delta}\left[\delta\left(\widetilde{x}_\delta(t) - \widetilde{x}_\Delta(t)\right)\right],$$

The scaled range: $$\mathrm{R/S}_\Delta(t) = \frac{R_\Delta(t)}{s_\Delta(t)}.$$

Hurst's amazing discovery

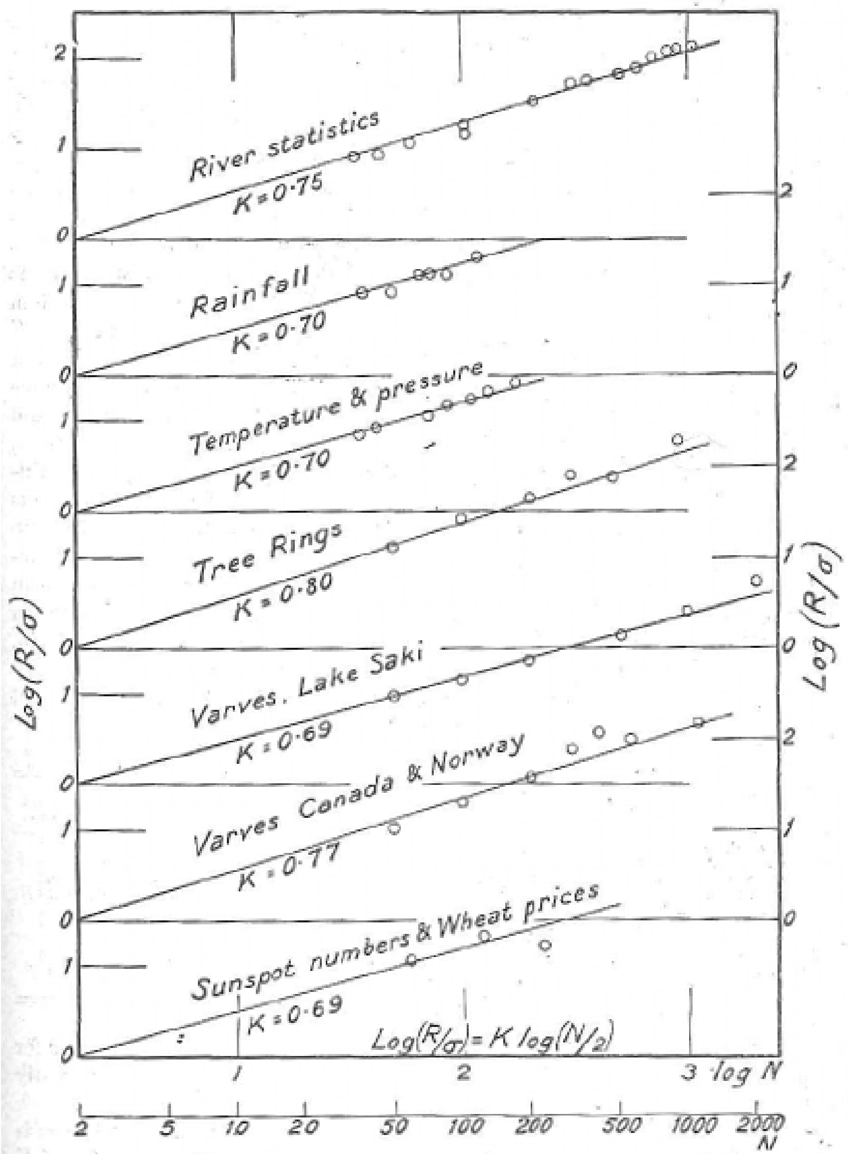
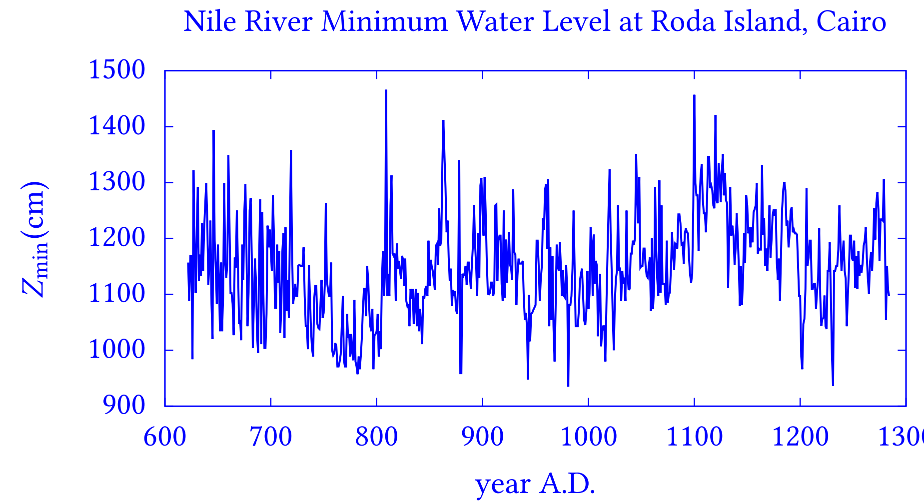


FIG. 4.—RELATION BETWEEN RANGE OF SUMMATION CURVE R, THE STANDARD DEVIATION σ, AND THE YEARS OF RECORD N



$$\text{Hurst's Law: } R/S(\Delta) = \left\langle \frac{R_\Delta(t)}{s_\Delta(t)} \right\rangle = c\Delta^H$$

From experiments with coins and sequences of independent random variables, it was expected to find $H = 0.5$. Instead, Hurst found $H = 0.72 > 0.50$ in geophysical time series.

We (now) know more than that:

Hurst's phenomenon **does not** have to do with independence of successive $x(t)'s$: variables serially correlated in time can (and do) exhibit $\Delta^{1/2}$ behavior.

The key to the problem appears to be the shape of the autocorrelation function. Let

$$\varrho(\Delta) \sim \Delta^{-p} \leftrightarrow S(\omega) \sim \omega^{p-1}$$

Then

$$1 < p < 2 \Rightarrow \text{“no Hurst” (fast decay with } \Delta)$$

$$0 < p < 1 \Rightarrow \text{“Hurst” (slow decay with } \Delta).$$

It is usually assumed that

$$p = 2 - 2H.$$

Just a little bit more theory

In the 1930's Taylor introduced the statiscal definition of the integral scale:

$$\mathscr{T} \equiv \int_0^\infty \varrho(\tau)\,\mathrm{d}\tau\,.$$

We have:

$\mathscr{T} < \infty$: No Hurst, R/S(Δ) $\sim \Delta^{1/2}$ [Feller, 1951].

$\mathscr{T} = \infty$: Hurst: R/S(Δ) $\sim \Delta^{H}$, $H > 1/2$.

$\mathscr{T} = 0$: Hurst (anti-persistence), R/S(Δ) $\sim \Delta^{H}$, $H < 1/2$. We will only discuss anti-persistent behavior very briefly in this talk.

The RMSE of $\widetilde{x}_\Delta$

In large part the above is empirical: an analytical result linking $\varrho(\Delta)$ to R/S(Δ) seems to be lacking. A result exists however for

$$\mathrm{RMSE}(\widetilde{x}_\Delta) = \left[\left\langle [\widetilde{x}_\Delta - \langle x \rangle]^2 \right\rangle\right]^{1/2} = \left[\frac{2\,\mathrm{Var}\{x\}}{\Delta}\int_0^\Delta \left(1 - \frac{\eta}{\Delta}\right)\varrho(\eta)\,\mathrm{d}\eta\right]^{1/2} :$$

$$\varrho \sim \Delta^{-p} \;\Rightarrow\; \mathrm{RMSE}(\widetilde{x}_\Delta) \sim \Delta^{-p/2} = \Delta^{H-1}.$$

When $\mathscr{T}$ exists, this reduces to Lumley and Panofsky's equation:

$$\mathrm{RMSE}(\widetilde{x}_\Delta) \approx \left[\frac{2\mathscr{T}}{\Delta}\mathrm{Var}\{x\}\right]^{1/2} \sim \Delta^{-1/2}.$$

Also, remember:

$$\mathrm{R/S}(\Delta) = \left\langle \frac{R_\Delta(t)}{s_\Delta(t)} \right\rangle \sim \Delta^H.$$

3. Example: Fractional Gaussian Noise

Fractional Brownian Motion and its "derivative", Fractional Gaussian Noise (FGN) (which does not exist in many classical senses ☺) appears to have been conceived by Kolmogorov, but was widely popularized by Benoît Mandelbrot. Here, we follow the latter.

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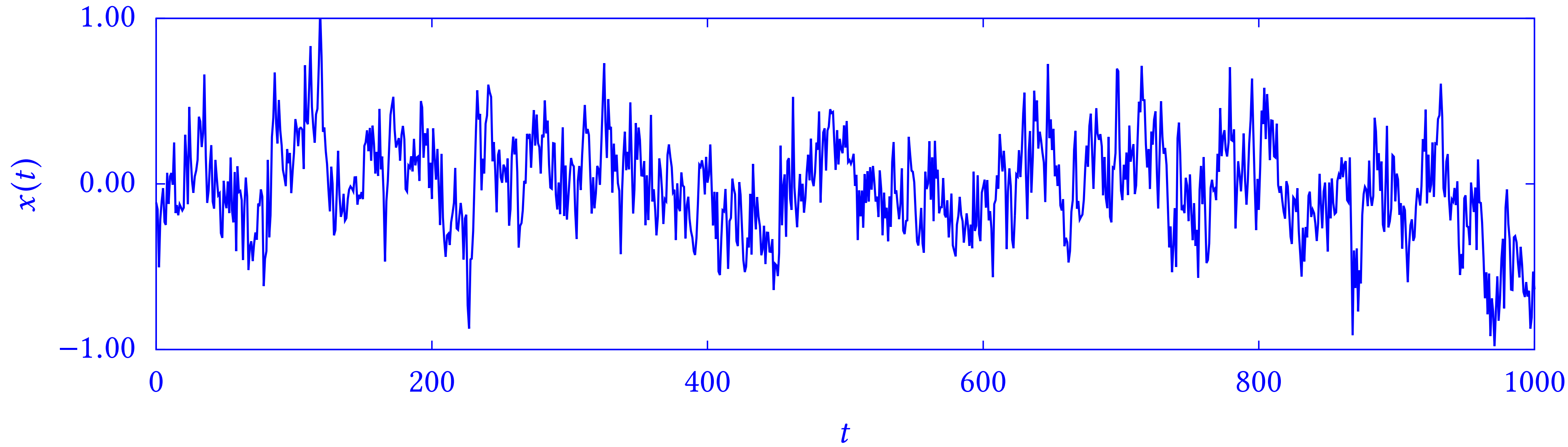
Here we generate FGN with Mandelbrot's **worst** model of **an approximation** to FGN (by his own account!), which is a moving average of the type

$$x(t) = Q_H G(t) + (H - 1/2)\sum_{k=1}^{M} k^{H-3/2} G(t-k), \qquad G(t) \sim N(0,1),\ \text{IID}.$$

One of the main reasons for the FGN model is to construct a stochastic model that does exhibit Hurst's phenomenon. Hurst's coefficient H is an explicit parameter of the model.

A simple example: 10,000 points of FGN with $H = 0.7$ ($p = 0.6$)

Objective: having a sample of FGN, how can we identify the value of H? First 1,000:



I am not assigning any particular meaning to the unit increment in the data.

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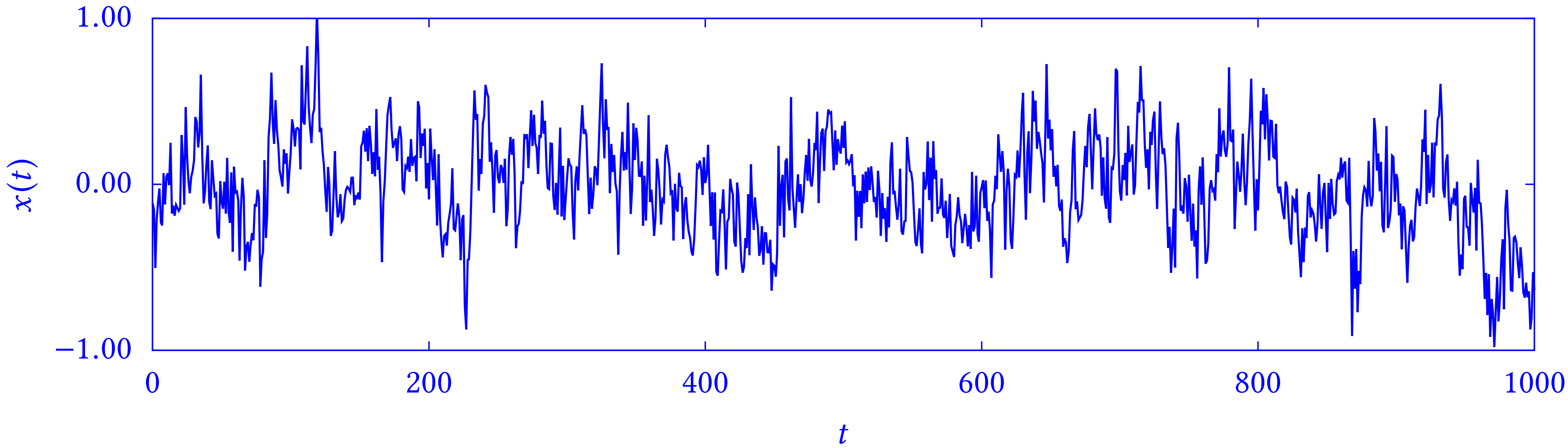
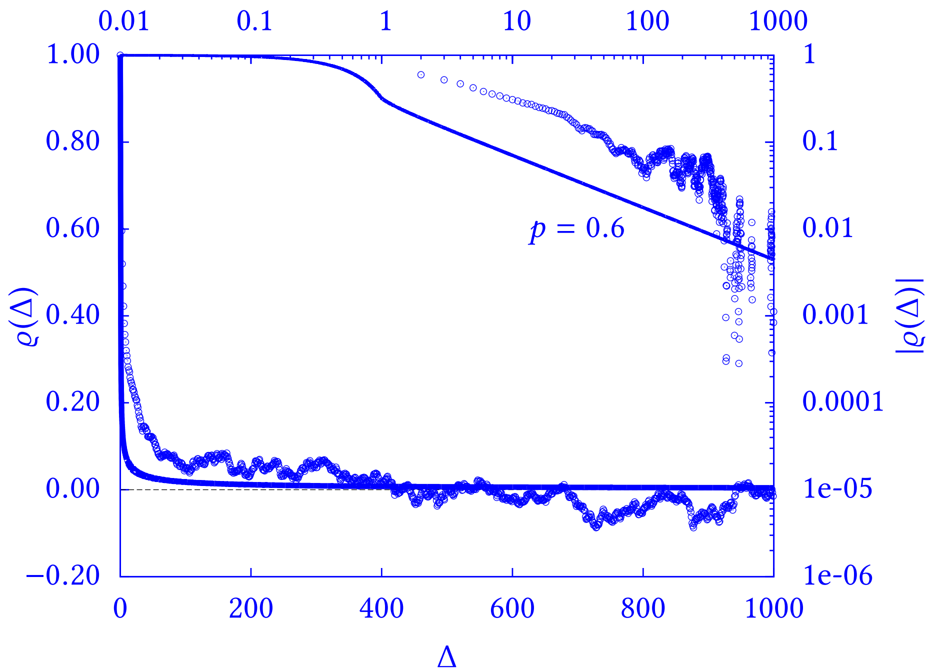


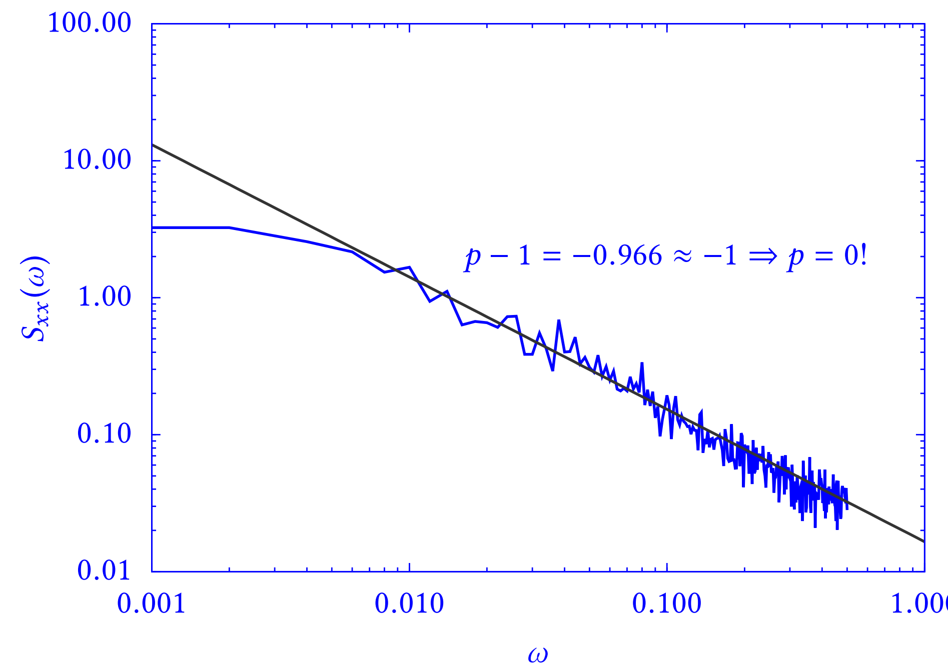
I am not assigning any particular meaning to the unit increment in the data.

Finally, some "turbulence"!

A micrometeorologist would...

Plot the autocorrelation and the spectrum (but H is elusive!):



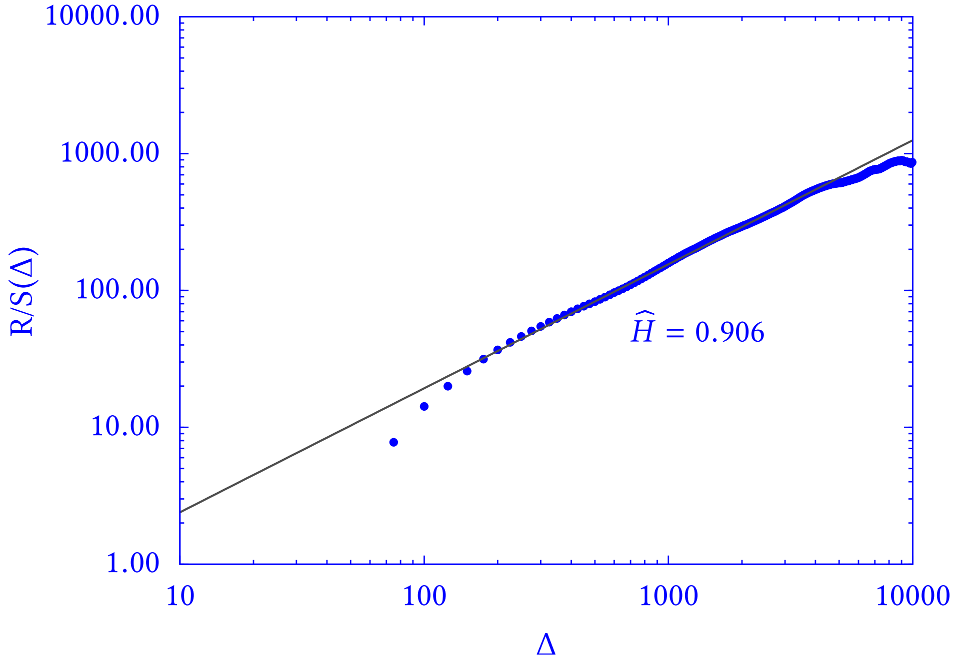


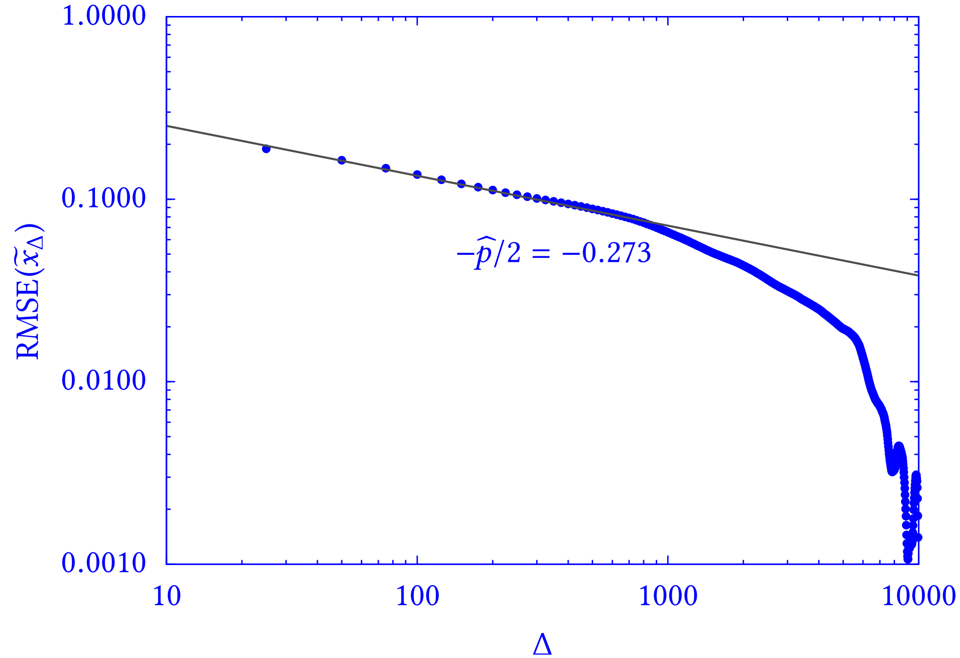
Here we have the theoretical $\varrho(\Delta)$:

$$\varrho(\Delta) = \frac{1}{2}\left[(\Delta+1)^{2H} - 2\Delta^{2H} + (\Delta-1)^{2H}\right] \sim \Delta^{2H-2} \text{ for large } \Delta.$$

R/S(Δ) and RMSE($\widetilde{x}_\Delta$)

Therefore, some of the most used statistical analyses in Micrometeorology are completely blind to Hurst's phenomenon: we need other statistics to detect it.





Interestingly, R/S(Δ), Hurst's original device that led to the discovery of Hurst's phenomenon, seems to be off the mark. RMSE($\widetilde{x}_\Delta$) is much better! (Only in this example?)

4. The Hurst Phenomenon in Turbulence

Here we give a brief history of people who identified the Hurst phenomenon explictly in Turbulence:

Early on, Sutton [1932] proposed that the autocorrelation function of wind components decays with the time-lag Δ as Δ^{-p}, with $p > 0$. Sutton obtained values of p equivalent to $H = 0.875$ and $H = 1$ in models of atmospheric diffusion. As noted by Taylor [1935], these values implied the nonexistence of $\mathscr{T}$.

Nordin et al. [1972] seem to have been the first to find the Hurst phenomenon in turbulent flows; their data came from laboratory flumes and measurements in the Missouri and Mississipi rivers.

The Hurst phenomenon was then identified in grid turbulence by Helland and van Atta [1978]; and in atmospheric concentration data by Gifford [1993].

Error estimates and the Hurst phenomenon

RMSE($\widetilde{x}_\Delta$) was already known to Mandelbrot and Wallis [1968]: but they subsequently chose to keep on analyzing data with R/S(Δ) (business as usual) instead.

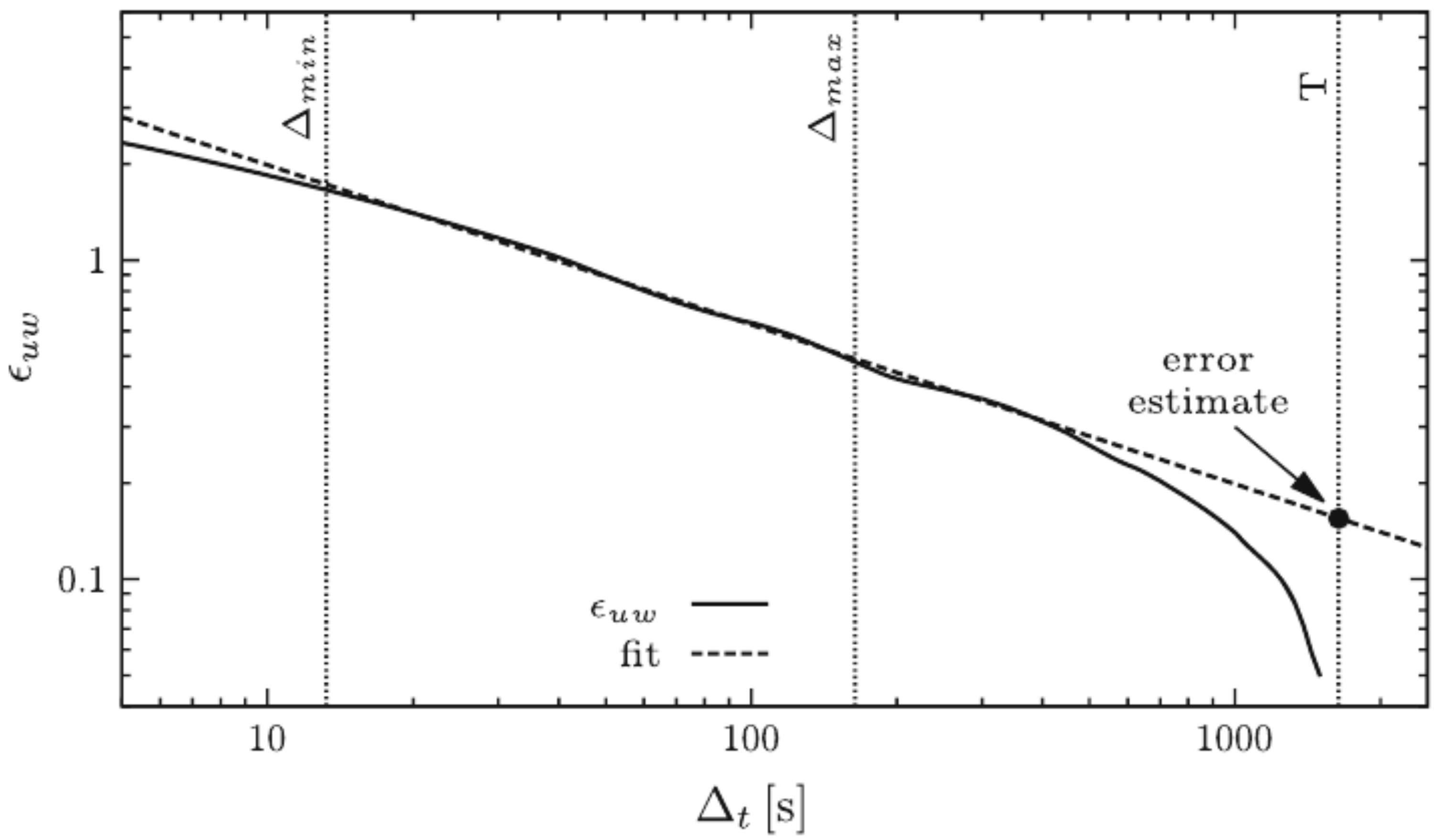
In the context of Hurst's phenomenon, it seems to have lain buried until Montanari et al. [1997]: here, they did put it to work to identify Hurst's phenomenom in Hydrology (again).

Recently, the power-law behavior of RMSE($\widetilde{x}_\Delta$) was independently proposed by Salesky et al. [2012] to calculate the random error of Surface-Layer turbulence statistics, as in

$$\widetilde{x}_\Delta = \frac{1}{\Delta}\int_0^\Delta [y'(t)z'(t)]\,\mathrm{d}t \Rightarrow \epsilon_{yz} = \mathrm{RMSE}\left(\left.\widetilde{y'z'}\right|_{\Delta=T}\right).$$

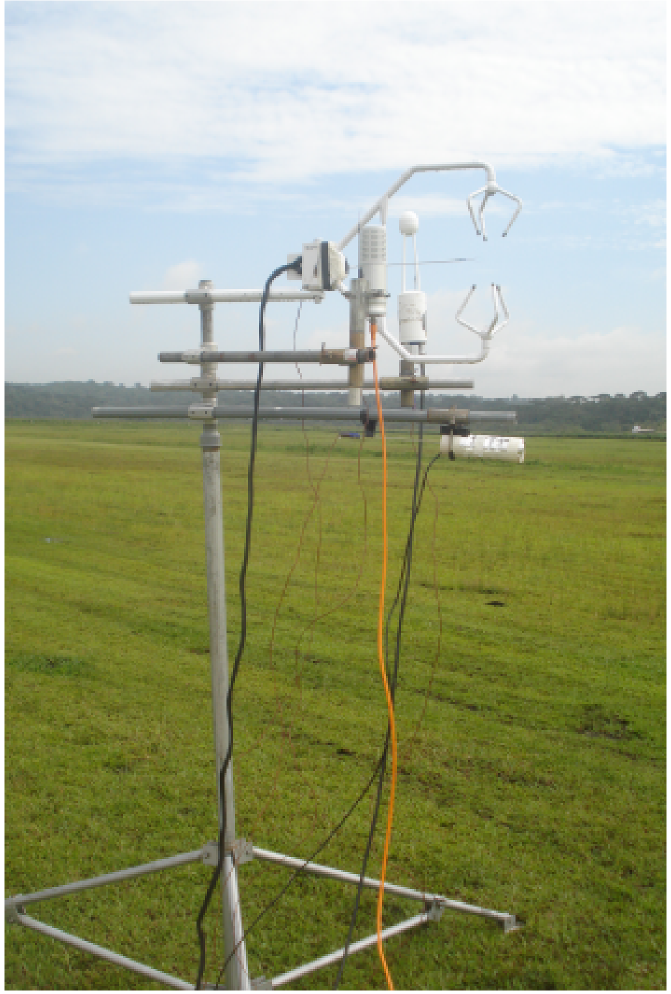
On the basis of Lumley and Panofsky's RMSE($\widetilde{x}_\Delta$) $\approx \left[\frac{2\mathscr{T}}{\Delta}\mathrm{Var}\{x\}\right]^{-1/2} \sim \Delta^{-1/2}$, Salesky et al. [2012] assumed $p = -1/2$ always, enforcing it even when data appeared to disagree. The present research started the same way, but the data insisted to disagree!

Eschew the integral time scale



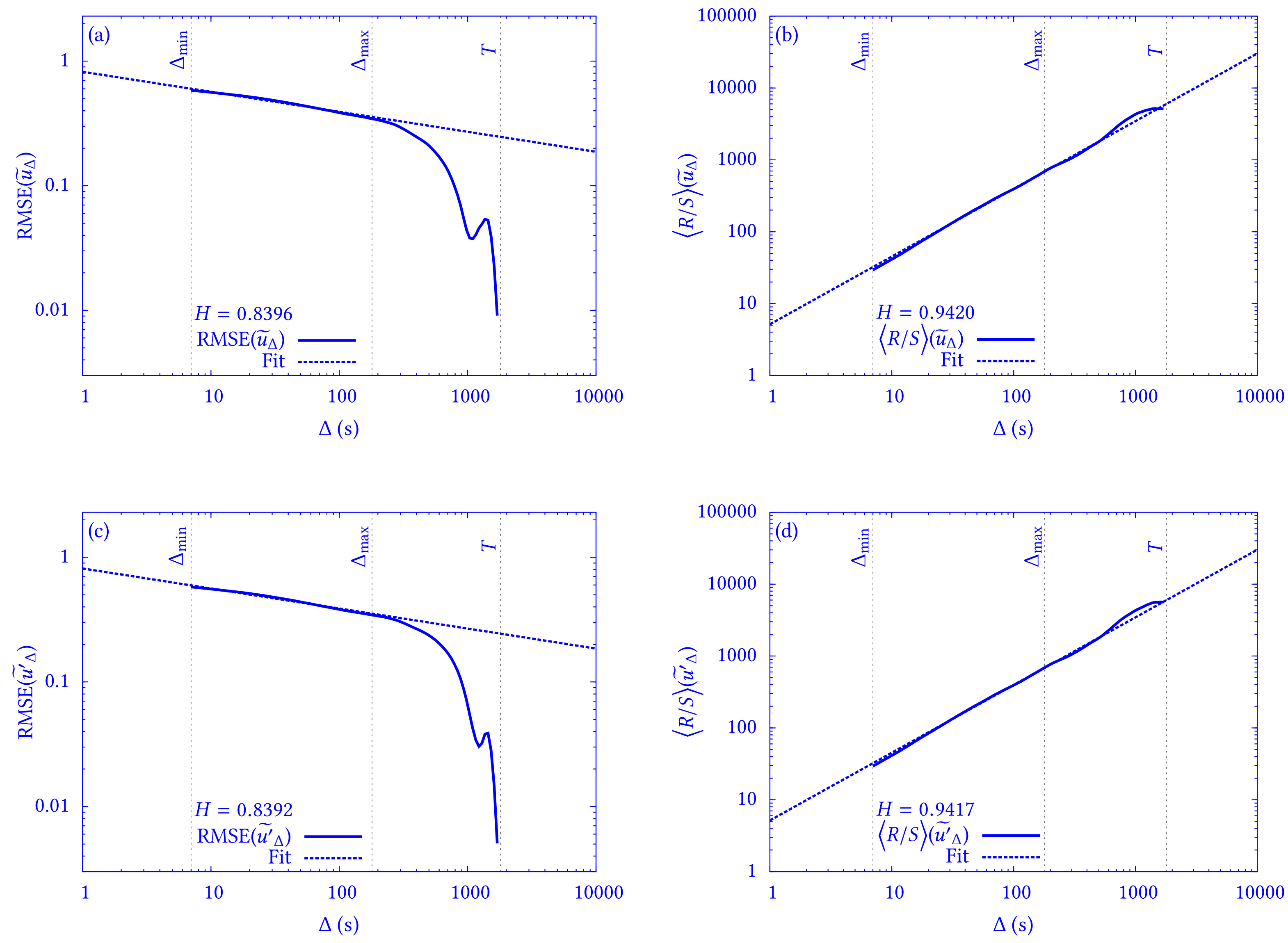
5. Surface-Layer Turbulence Data and Hurst

Two data sets: Tijucas do Sul (short grass) and Missal (Itaipu Lake).



Most analyses were made after linear detrending.

Linear detrending does not affect significantly the estimates of H.



Data analysis

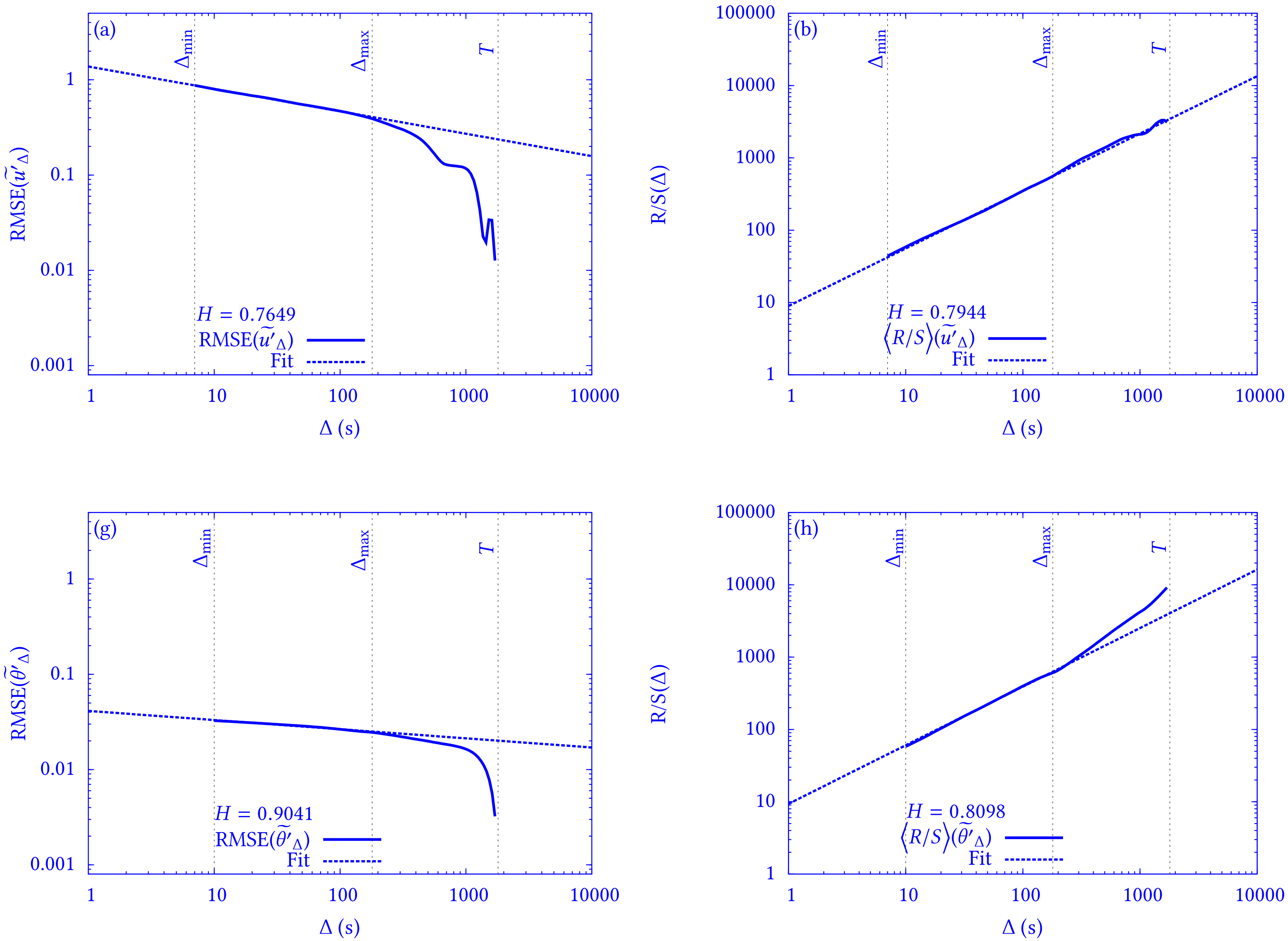
So we did the obvious (in **hindsight**!): we allowed Surface-Layer Turbulence to exhhibit Hurst's phenomenon (but we did not, by any means, ordered it to!).

The goal is simple: just to re-do Salesky et al. [2012]'s approach, but letting $p \neq 1$.

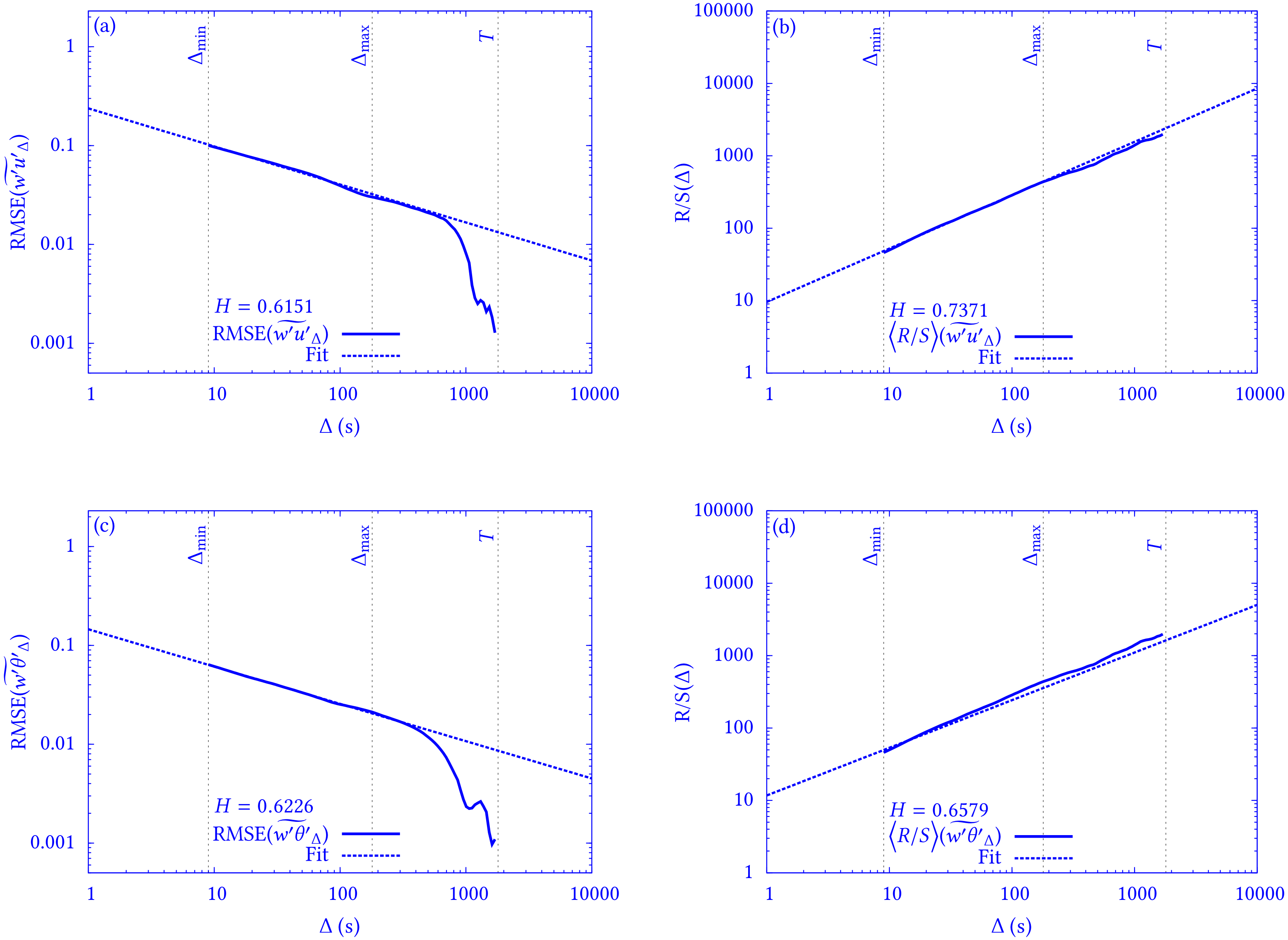
All data sets analyzed display Hurst's phenomenon very distinctively.

Hurst's phenomenon is stronger (larger H) for first-order data. It is still present for second-order data.

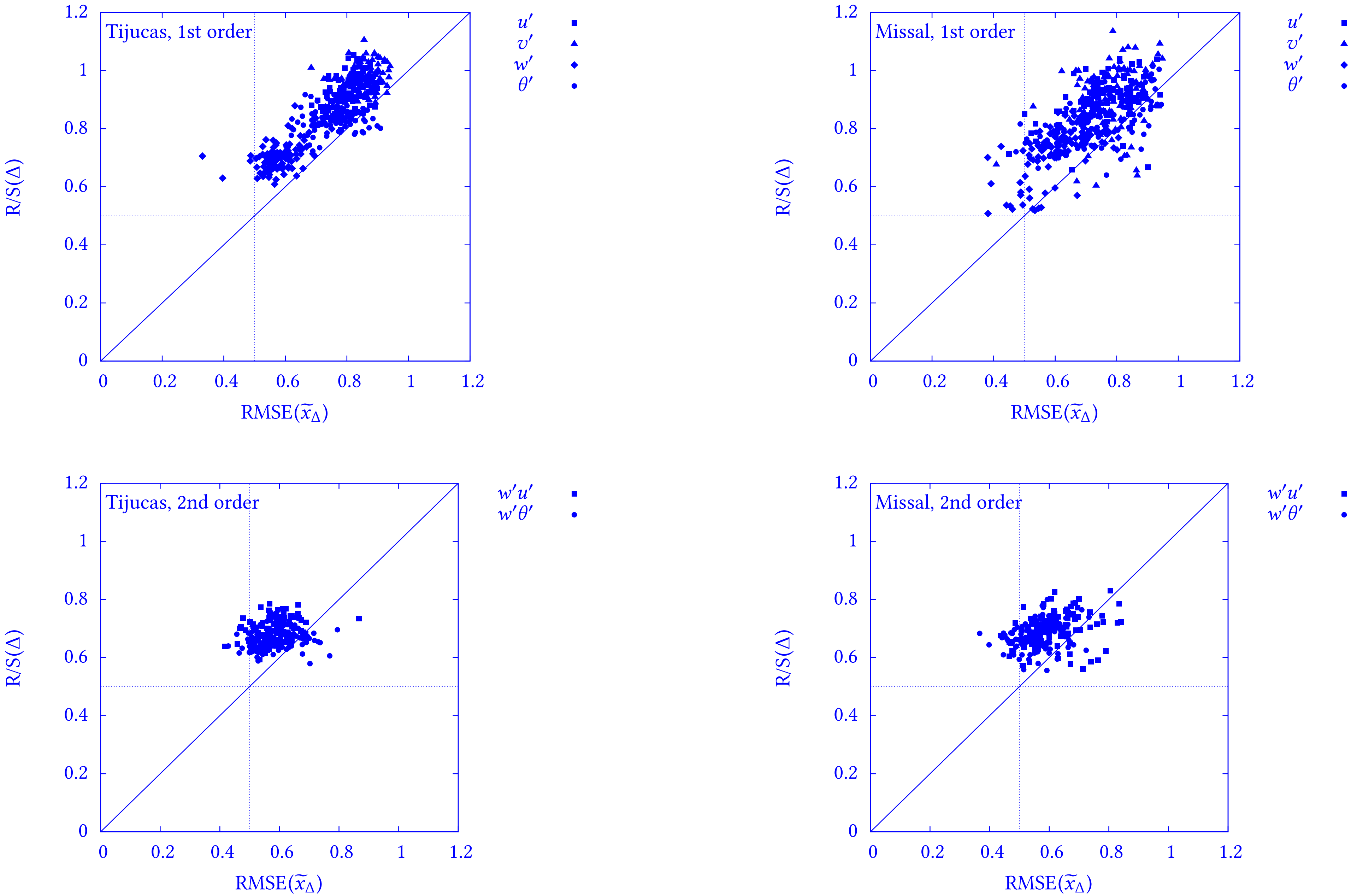
Missal data, 1$\underline{^{\text{st}}}$ order



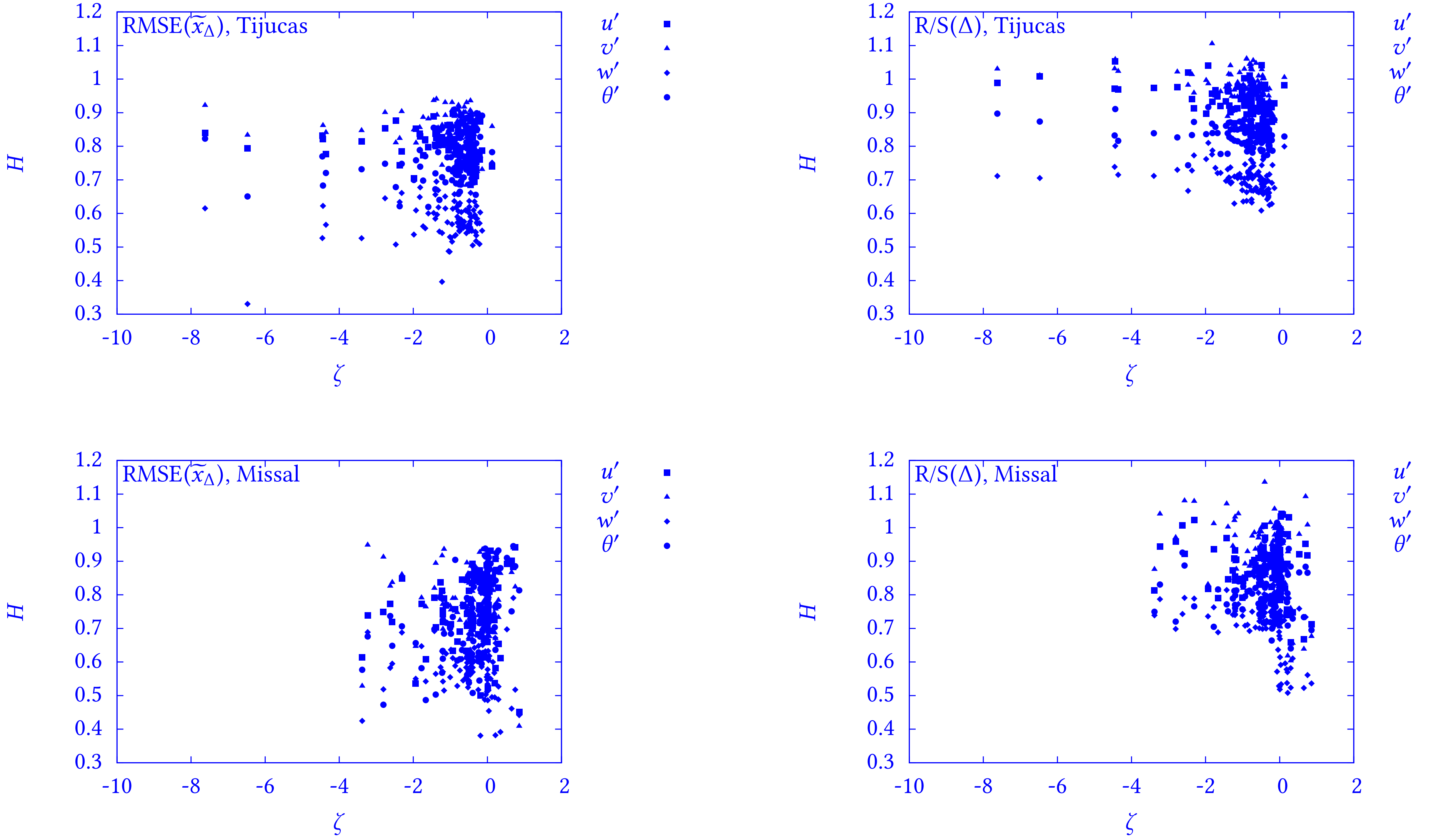
Tijucas data, 2$^{\underline{nd}}$ order



Is R/S(Δ) a biased estimator of H?



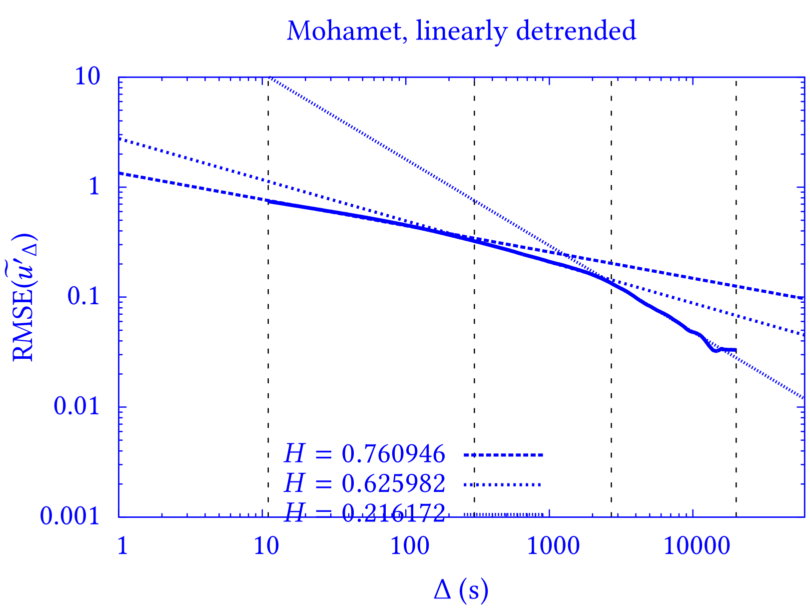
The Hurst phenomenon is probably outside the scope of MOST

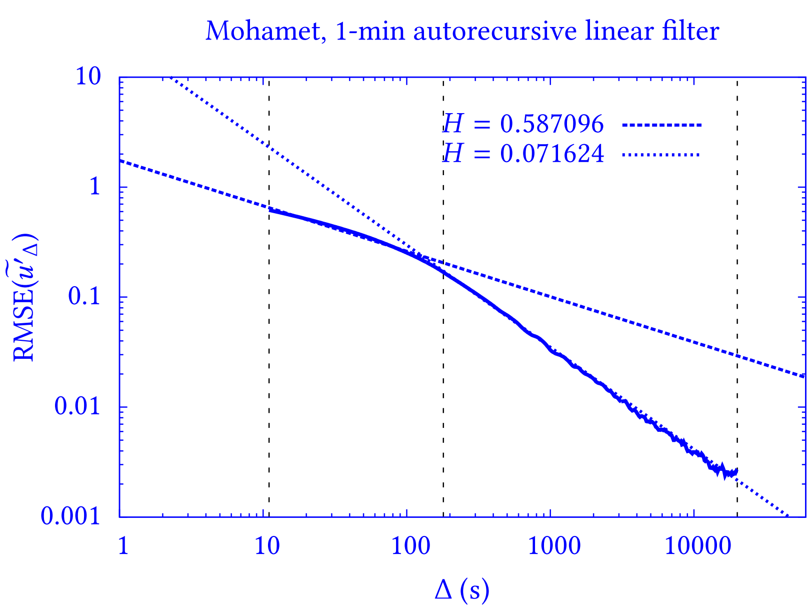


(Very long range, very low frequencies)

Filtering may alter H significantly!

Look at Mohamet's u and θ_v data [Chamecki, 2013]: 07:30 h of near-steady, near neutral turbulence (15 times as long as "standard" runs).





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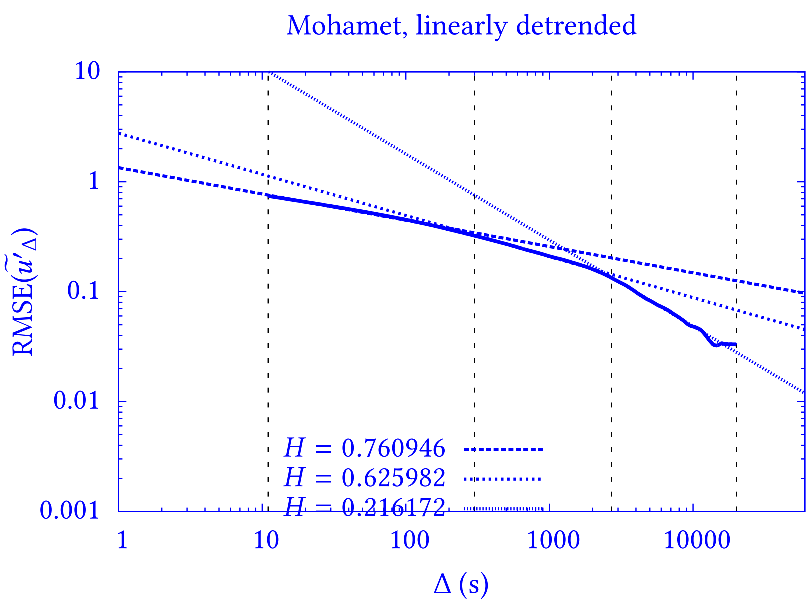

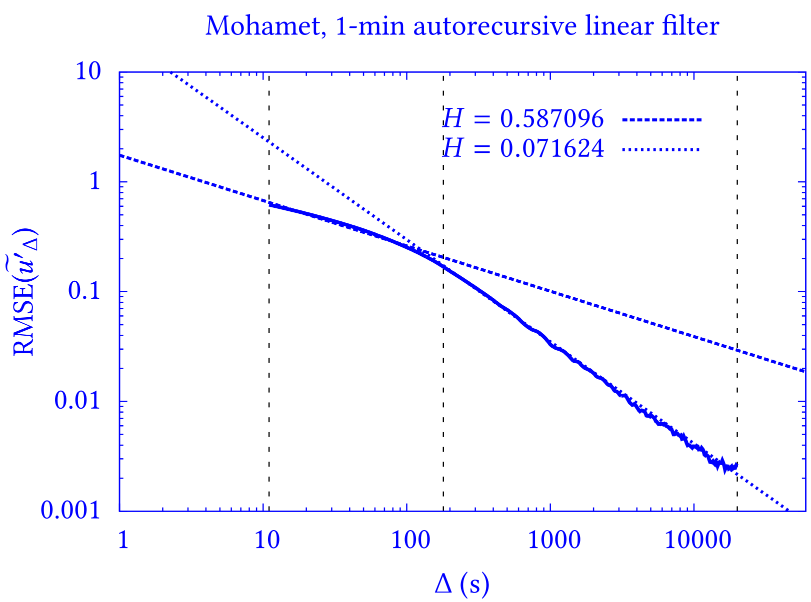

⇒ Filtering is not recommended when calculating "Hurst statistics".

Conclusions

- Hurst's phenomenon is ripe in surface-layer turbulence.
- Tools devised to "see" inertial-range behavior are blind to Hurst's phenomenon.
- Taylor's integral time scale $\mathscr{T}$ often does not exist in surface-layer turbulence.
- R/S(Δ) and RMSE($\widetilde{x}_\Delta$) are **different** estimators (they don't yield the same H).
- This does not prevent error estimates from being possible, but errors may be somewhat larger than we thought, because of the lower decay of RMSE($\widetilde{x}_\Delta$) with Δ.
- The Hurst phenomenon is (very likely) outside the scope of Monin-Obukhov Similarity Theory. This is expected, due to the very long-range nature of Hurst's phenomenon.
- Filtering can alter Hurst statistics dramatically. Avoid it by all means when looking for Hurst's phenomenon in surface-layer turbulence.

Many thanks

…for your attention!

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